The Screening of Nuclear Reactions: Astrophysics, Experiments, Laboratory Plasmas*

Nuclear Astrophysics and the National Ignition Facility

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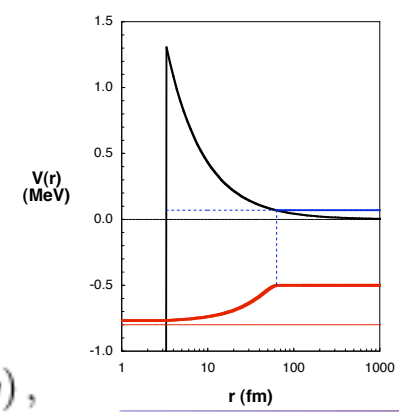
Many Body Plasma Screening has a Significant Effect on Two Body Fusion Reactions of Astrophysical and Laboratory Interest

- Reactions of Interest include: the pp chain driving our Sun, particularly neutrino producing branches , CNO cycle, d + t → α + n, d + d → p + t (n+^{3}He), etc.
- The physical picture begins with the Gamow semi-classical barrier penetration analysis of fusion:

$$\sigma\left(E_{CM} << E_{barrier}\right) \approx \frac{1}{E_{CM}} \exp\left[-\sqrt{\frac{E_S}{E_{CM}}}\right] \times \text{nuclear terms} \times \text{correction terms}$$

- The exponential term arises from the standard WKB form exp-(∫ pdr), with the integral taken between the barrier turning points.
- Screening is due to the many body modification of the basic Coulomb potential.
- In a weakly coupled plasma in thermal equilibrium, this is tantamount to the potential shift near the origin due to Debye screening → Salpeter screening:

$$D^{-2} = \sum_a \frac{4\pi\, e_a^2\, n_a}{kT}, \qquad f_0 = \exp\left(\frac{Z_1 Z_2 e^2}{kTD}\right) = \exp\left(Z_1 Z_2 g\right),$$



‘Astrophysical’ S factor S(E): Nuclear Effects, Corrections to WKB, etc.

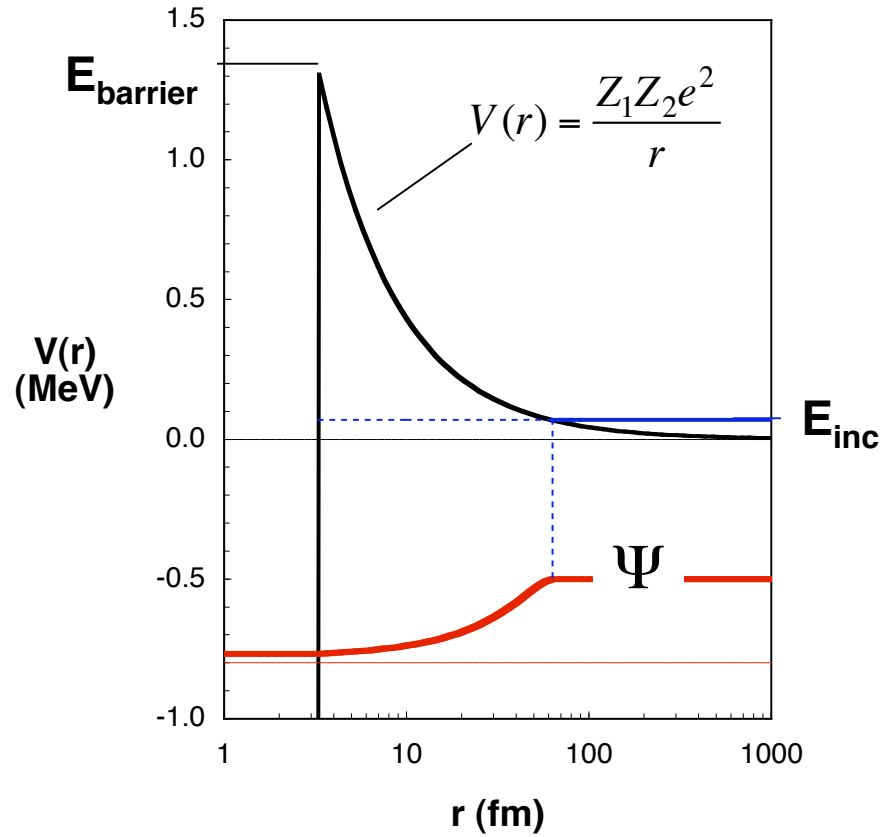
$$\frac{\Psi_{inside}}{\Psi_{outside}} \approx \exp\left[-\sqrt{\frac{E_S}{E_{CM}}}\right]$$

where $$E_S \equiv 2\left(\pi\alpha Z_1 Z_2\right)^2 \times \mu c^2$$

$$\approx Z_1^2 Z_2^2 A_{red} \times 1 \text{ MeV/AMU}$$

Sommerfeld-Gamow Energy

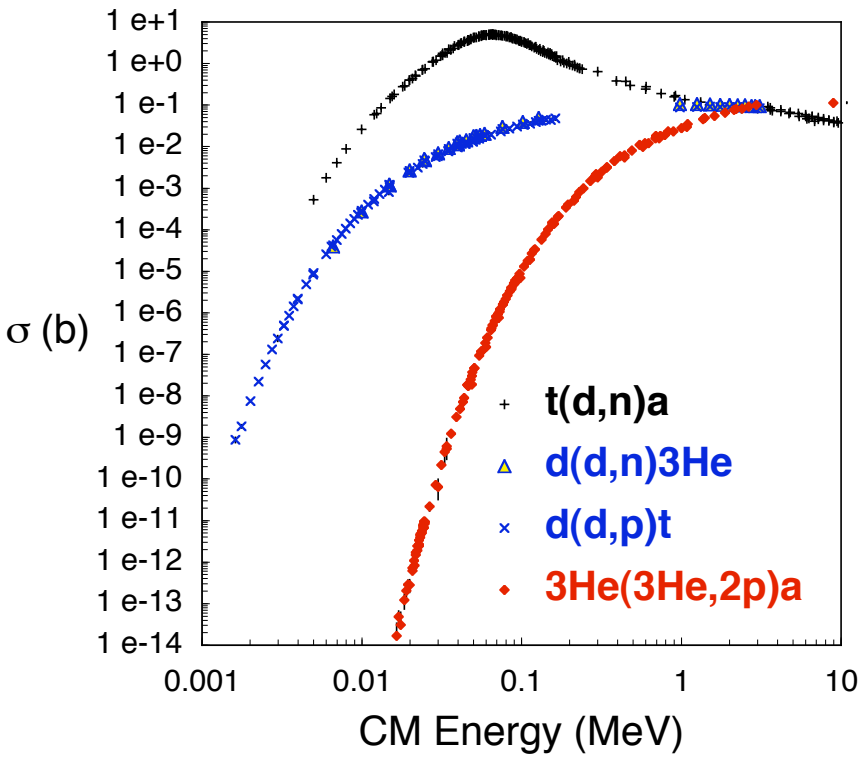
‘Astrophysical’ S(E) factor (can depend on Breit-Wigner resonances, other nuclear physics details, corrections to semi-classical). A few cases, e.g. pp → d + $e^+ + \nu_e$, S(0) known.

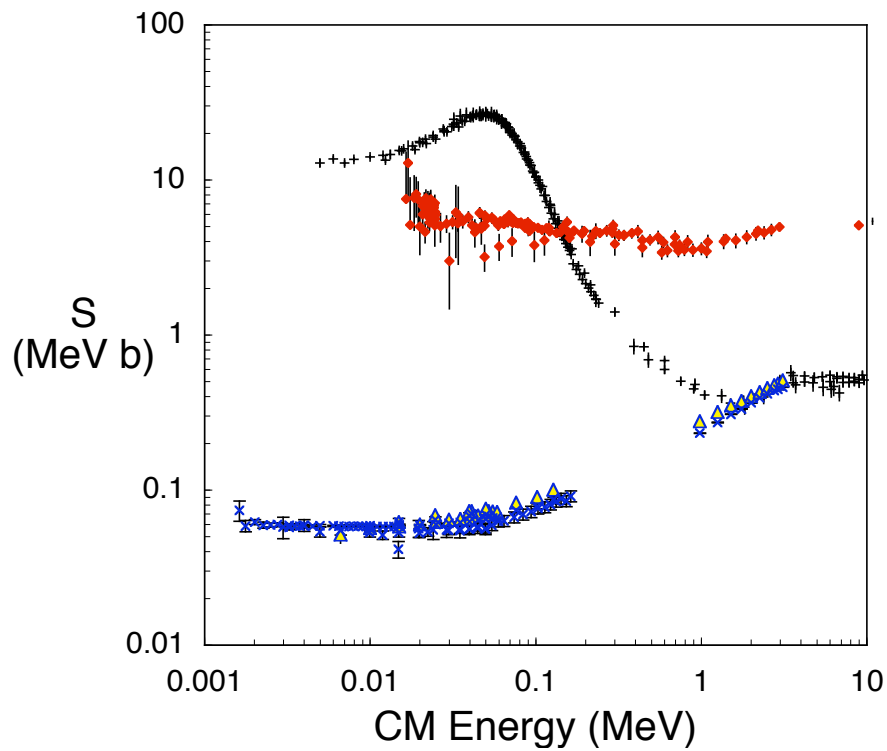


$$\sigma\left(E_{CM} << E_{barrier}\right) \approx \frac{1}{E_{CM}} \exp\left[-\sqrt{\frac{E_S}{E_{CM}}}\right] \times \text{nuclear terms} \times \text{correction terms}$$

S(E)

Raw Cross Sections Versus Astrophysical S factor





‘Semiclassical’ Weighted Thermal Average of Cross Sections - Reaction Rates

- Reaction rate per unit volume is appropriate thermal average of σv
- In thermal equilibrium the Maxwell-Boltzmann average of σv is

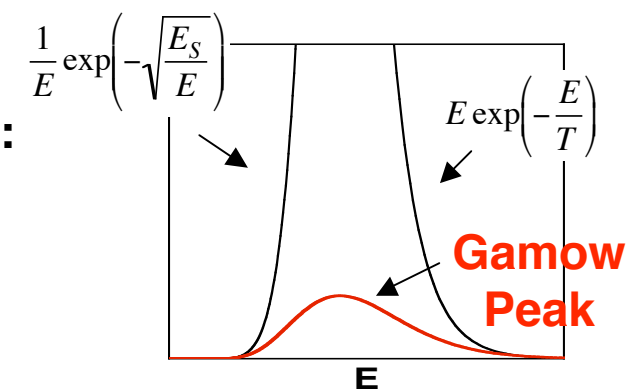
$$\langle \sigma v \rangle = \sqrt{\frac{8}{\pi \mu c^2}} \times T^{-3/2} \times \int_0^\infty E\sigma(E) e^{-E/T} dE$$

- Assuming that S(E) varies slowly, the integrand is peaked at ‘Gamow’ energy

$$E_G = \left[\tfrac{1}{4} E_S T\right]^{1/3}$$

- Overall reaction rate scales with Gamow energy as:

$$\langle \sigma v \rangle \approx \left[\frac{8}{3}\frac{E_S}{\mu c^2}\exp\left(-\sqrt{\frac{E_S}{E_G}}\right)\right]^{1/2} \times \sigma(E_G) c$$



- Note that this average assumes that phase space variables (momentum & positions of particles) are sharply defined. Only an approximation to QM.

Standard Solar Model – pp chain and basic branches

- **Standard Solar Model (SSM)**
  - **J.N. Bachall and R.N. Ulrich, Rev. Mod, Phys. 60, 297 (1988)**
  - **–The *pp*-cycle in the Sun "burns" protons to make Helium**
  - **–Most of the verification comes through the detection of Solar neutrinos and helio-seismology**
  - **–Neutrino oscillations (masses and mixing angles) could be better constrained by better knowledge of the nuclear reaction rates: $^7Be(p,\gamma)^8B$ and $^3He(^4He,\gamma)^7Be$**
  - **– current phenomenological theory can't predict resonances, S(E) for $^3He(^4He,\gamma)^7Be$ varies by 50% (but soon within computational reach?)**
- **What is the role of the CNO cycle?**

$pp \rightarrow {}^2H + e^+ + \nu_e$		
${}^2H + p \rightarrow {}^3He + \gamma$		
${}^3He + {}^3He \rightarrow {}^4He + 2p$	85%	
${}^3He + {}^4He \rightarrow {}^7Be + \gamma$	15%	
$e^- + {}^7Be \rightarrow {}^7Li + \nu_e$		
${}^7Li + p \rightarrow 2\,{}^4He$		
$p + {}^7Be \rightarrow {}^8B + \gamma$	0.02%	
${}^8B \rightarrow {}^8Be^* + e^+ + \nu_e$		
${}^8Be^* \rightarrow 2\,{}^4He$		

- **Many S-factors rely on extrapolated measurements**

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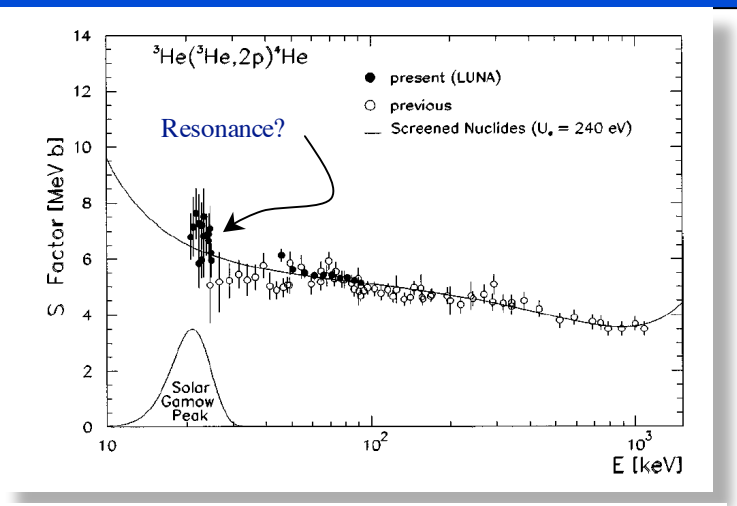
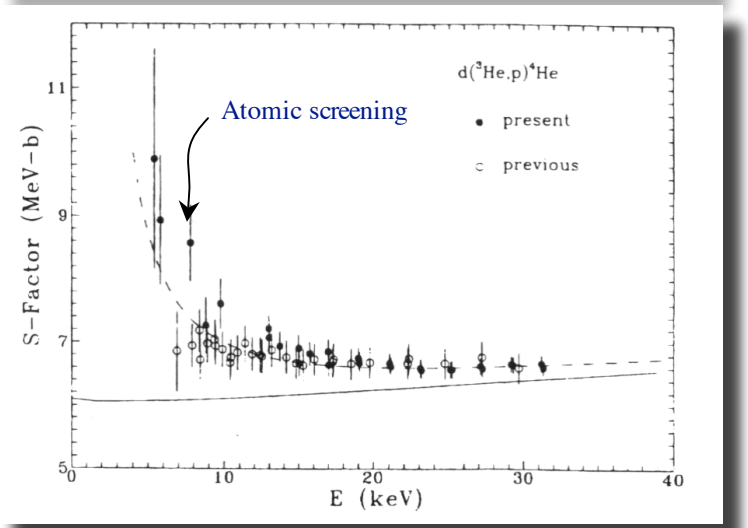
S-factors are difficult to measure experimentally

- Rates are low
  - –sensitive to backgrounds
- Solar energies are generally lower than can be achieved in standard experiments
  - – Often must extrapolate to useful energy region
  - – Heroic experiments (LUNA) have given very important data at the Gamow peak.
- Screening to atomic electrons is large and must be unfolded (part of our screening story).
- Breit - Wigner Resonances (unknown) can be important - think dt
- Theory is needed to supplement experiments.
- But, no fundamental theory exists (we are making progress, though!)





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Salpeter Screening - Debye Length is the Key

- Salpeter (1954) screening takes into account the reduction of the barrier potential due to (static) Debye screening.

$$\phi_1 = (Z_1 e/r)\exp(-r/D) \qquad D^{-2} = \sum_a \frac{4\pi\, e_a^2\, n_a}{kT}$$

- For the Sun's center T ~ 1 keV & ρ ~ 100 gr/cc (with solar Z mix) → D ~ .3 Å and the plasma coupling parameter $e^{2/}$ D kT ~ .04.
- The quasi-static potential (of ion Z_1) seen by a fusing ion Z_2 is shifted down near the origin by the constant $Z_1 e/D$ (Taylor expand), resulting in an exponential rate enhancement of:

$$f_0 = \exp\left(\frac{Z_1 Z_2 e^2}{kTD}\right) = \exp\left(Z_1 Z_2 g\right),$$

- Under these conditions, the pp reaction is enhanced by 5%, and the p+^{7}Be reaction is enhanced by 20%.

- Possible criticisms include: weak coupling, possible dynamical effects causing ion correlations, quantum correlations in screening expectation value, application to non-equilibrium (I.e. multi T plasmas).

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Many Body Analysis of Plasma Screening in Thermal Equilibrium - Quantum & Dense Plasma Effects (1)

- Brown and Sawyer have treated plasma screening including thermal and quantum effects on the same footing: includes electrons & ions, degeneracy corrections, no assumption of 'factorization' of potential and barrier penetration, includes entanglement of initial and final states.
- Assumptions, plasma weak coupling and quantum perturbation theory, high Q reaction, no feedback of reaction on average thermal state of plasma (reactions are rare), linear response.

$$\mathcal{H}(\mathbf{r},t)=\mathcal{K}(\mathbf{r},t)+\mathcal{K}^{\dagger}(\mathbf{r},t),$$

in which

$$\mathcal{K}(\mathbf{r},t)=ge^{-iQt}\psi_4^{\dagger}(\mathbf{r},t)\psi_3^{\dagger}(\mathbf{r},t)\psi_2(\mathbf{r},t)\psi_1(\mathbf{r},t)$$

$$\Gamma=-i\int_{-\infty}^{0}dt\int(d\mathbf{r})\langle[\dot{n}(0),\mathcal{H}(\mathbf{r},t)]\rangle_{\beta}$$

$$=-\int_{-\infty}^{0}dt\int(d\mathbf{r})\langle[\mathcal{K}(0)-\mathcal{K}^{\dagger}(0),\mathcal{K}(\mathbf{r},t)+\mathcal{K}^{\dagger}(\mathbf{r},t)]\rangle_{\beta},$$

$$\Gamma=g^2\exp\{\beta(\mu_1+\mu_2)\}\int_{-\infty}^{+\infty}dt\int(d\mathbf{r})e^{iQt}$$
$$\times\langle\langle\mathbf{r}_1'=0=\mathbf{r}_2',-i\beta|U(\beta)V(t)|\mathbf{r}_1'=\mathbf{r}=\mathbf{r}_2',t\rangle$$
$$\times\langle\mathbf{r}_3'=\mathbf{r}=\mathbf{r}_4',t|W(t)|\mathbf{r}_3'=0=\mathbf{r}_4'\rangle\rangle_{\beta}.$$

$$\Gamma_P=\left\{1+\beta e_1e_2\left[\kappa_D-\frac{1}{2a_0}\frac{1}{e^{-\beta\mu_e}+1}\right]\right\}\Gamma_C,$$

$$\left[\kappa_D-\frac{1}{8}\kappa_{D,e}^2\lambda_e\right].$$

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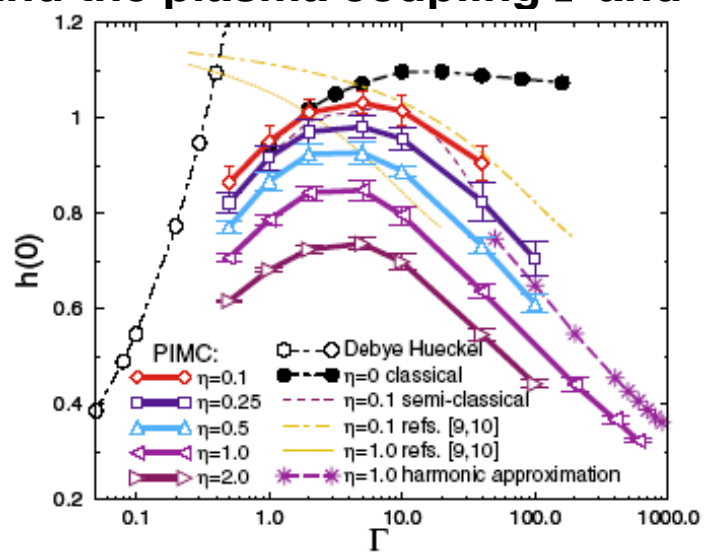
Many Body Analysis of Plasma Screening in Thermal Equilibrium - Quantum & Dense Plasma Effects (2)

- Dense plasma effects on the short distance behavior of the pair correlation function has been studied using Path Integral Monte Carlo for the quantum "One Component Plasma" (uniform electron neutralizing background, mobile ions) - Pollock & Militzer 2003.
- Assumes that reaction rate is of semiclassical average form: $R \sim n_2 \langle \sigma (v_{12}) j_{12}(r=0) \rangle$ and studies $j_{12}(r=0) = n_1 g_{12} v_{12}$ in terms of the many body corrections to the pair correlation function g_{12} and the plasma coupling Γ and 'quantumness'.

$$ng(r) = \left\langle \frac{1}{N} \sum_{i \neq j} \delta(r - r_{ij}) \right\rangle \equiv n g_{\mathrm{bin}}(r)\, e^{H(r)}$$

$$\Gamma h(r) \equiv H(r).$$



- Result is quantum 'smearing' reduction.

‘Atomic’ (Bound State) Screening in Astrophysics and Experiment

- So far, we have been discussing screening in either weak coupling (plasma) perturbation theory, or in Monte Carlo treatments of OCP-type models. These intrinsically leave out atomic bound states. These are relevant both to laboratory scattering experiments as well as higher Z ions in plasmas.

- Physically this is tantamount to an additional potential reduction (beyond that due to continuum plasma electrons and ions) U_e. When $U_e << E_{cm}$ the screening enhancement goes like: $\exp(\pi(E_S/E_{cm})^{1/2} U_e/E_{cm})$. For the case of $^3He(d,p)^4He$ this required $U_e \sim 186$ eV at $E_{cm} \sim 5.88$ keV.

- The (adiabatic) effect of bound atomic electrons has a straightforward Yukawa parameterization:

$$V_{electrons} = \frac{Z_2 e^2}{r}\left[\left(1+\frac{r}{2r_0}\right)\exp\left(-\frac{r}{r_0}\right)-1\right],$$

$$\text{where } r_0 \equiv \frac{a_0}{2(Z_1+Z_2)}$$

$$V_{electrons}(r << r_0) \Rightarrow \frac{(Z_1+Z_2)Z_2 e^2}{a_0}$$

- Example: Under solar core conditions, either Saha, or ACTEX (F. Rogers) predicts ~ 14% occupation of the 1s in Be! Therefore there is an additional screening enhancement of $^7Be(p,\gamma)^8B \sim 1$-2%

Are there Significant Plasma Screening Effects on Thermonuclear Reactions Beyond 'Salpeter' Screening?

- **The Salpeter correction is essentially a Debye screening correction to the Gamow barrier penetration factor for fusion:**

$$\Gamma_{Salpeter} = \Gamma_0 \exp(e_1 e_2 \kappa_D \beta)$$

$$\kappa^2_D = \sum 4\pi (Z_s e)^2 \beta \langle n_s \rangle_\beta$$

- **In the Sun this yields ~ 5% enhancement for the pp reaction and 20% for p(^{7}Be,γ)^{8}B**

- **Quantum (off shell) and multi T corrections:**
  - **We have seen that equilibrium quantum corrections are small (electron quantum reduces Salpeter effect by ~10%, ions by < 1%) (Brown & Sawyer):**

$$\Delta\Gamma \sim -\Gamma_0 (\beta e_1 e_2 \kappa_D^2 \lambda_e)/8$$

  - **Non-equilibrium effects argued large (Savchenko et. al.), multi T (there reactions DO have dynamical feedback on plasma!), an open question, though when this occurs, T_{ion} and $T_{electron}$ are BIG and the overall effect is likely to be modest:**

$$f(p) = f_{MB} + \frac{h\nu_p T}{2\pi \varepsilon_p^2} e^{\frac{\mu}{T}} \qquad \sigma_{ij}(\varepsilon_p) = \frac{S_{ij}}{\varepsilon_p} e^{-\pi(\varepsilon_G/\varepsilon_p)^{1/2}}$$

$$\langle \sigma_{ij} v \rangle_f / \langle \sigma_{ij} v \rangle_{f_{MB}}$$

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Stellar Astrophysics at NIF: Measurements of Basic Thermonuclear Reactions

- Assume Thermonuclear Reaction Rates between species i and j are of the form:

$$R \sim \langle \sigma_{ij} v \rangle_f \qquad \sigma_{ij} = \frac{S_{ij}(\varepsilon_p)}{\varepsilon_p} e^{-\pi(\varepsilon_G/\varepsilon_p)^{1/2}}$$

- Use S factors are extrapolated to the relevant stellar Gamow 'weighting' regions from higher energy experimental data[1] – laboratory 'cold' electron screening effects are significant
- Thermonuclear reactions can be observed in 'passive' NIF implosions or as by products of the temperature runaway in d + t burn -but measurement challenging!

$^3He(^3He,2p)\alpha$	~ 10^8 reactions at 8 keV in 20 ps (10^{17} initial nuclei, note 'targets')
$^7Be(p,\gamma)^8B$	~ 4 x 10^4 reactions
$^3He(\alpha,\gamma)\,^7Be$	~ 3 x 10^5 reactions
$^{15}N(p,\alpha)^{12}C$	~ 6 x 10^6 reactions

- Plasma screening scales from Solar case as ~ $(n/T^3)^{1/2}$ - therefore typically smaller than Sun
- What about Multi T case when ignition occurs?

[1]Adelberger et. Al., RMP 70, (1998), 1265

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