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# Kinetic effects in a fluid plasma transport approximation and some recent simulation results

presented by E. Vold

with acknowledgement to many: K. Molvig, A. Simakov,

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# Introduction

- **Kinetics includes all collisional effects.**
  - **momentum and energy exchange in:**
    - **thermal conduction & equilibration,**
    - **viscosity,**
    - **species particle flux relative to $U_{cm}$**

*Approximated as diffusive processes with characteristic scale length, $L_D$:*

$$L_D \sim 2[D\,t]^{1/2} \sim [D_{ij} \sim \eta \sim \kappa \sim T^{5/2}]t^{1/2}$$

$$L_D \sim [T^{5/2}t]^{1/2} \sim T^{5/4}t^{1/2}$$

- **Multi-species fluid transport approximations can represent some kinetic effects.**
  - **assumed 'nearly' Maxwellian distributions**
  - **small parameter Kn:**
    - **MFP < gradient scales**
    - **t >> $\tau_{ij}$**

$$Kn \sim \lambda / L \xrightarrow{diffusion} \lambda / L_D \sim \frac{\lambda}{2(D\,t)^{1/2}} = \frac{\lambda}{2((v_{th}\lambda t)^{1/2}} = \frac{v_{th}\tau_{ij}}{2((v_{th}^2\tau_{ij}t)^{1/2}} = \frac{1}{2}\left(\frac{\tau_{ij}}{t}\right)^{1/2}$$

- **ICF Shock heating at interfaces:**
  - **fluid approximations may not be formally valid but may perform reasonably well.**

# Kinetics to plasma fluid transport **

## self-consistent plasma particle, heat and momentum flux

## ** condensed from: Simakov and Molvig, PoP 2016

- **Boltzmann equation:**

$$\frac{\partial f_i}{\partial t} + \nabla \cdot (v_i f_i) + \nabla_v \cdot (a_i f_i) = C_{ij}$$

- **Integrate over velocities: LHS, first order in Kn -> 'fluid terms', RHS -> 'collisions', consider flux vector.**
- **Approximate distribution function w/ Legendre (Sonine) polynomials (for vector piece of solution):**

$$f_i \sim (f_{Max-i} + f_i^1) \sim (f_{Max-i} + \sum_k \alpha_i^k L_k^{3/2}) \sim (f_{Max-i} + \alpha_i^0 L_0^{3/2} + \alpha_i^1 L_1^{3/2} + \alpha_i^2 L_2^{3/2})$$

- **Solve coupled equations for particle and heat flux coefficients of each species, i, of N species:**

$$\left[ v_{ii} + \sum_j v_{ij} \right]_i^{3x3} [\alpha^o, \alpha^1, \alpha^2]_i - \sum_j \left[ v_{ji} \right]_i^{3x3} [\alpha^o, \alpha^1, \alpha^2]_j = D_i \sim [\frac{d_i}{\chi_i}, -\frac{5}{2} \nabla \log T_i, 0]_i$$

- **Binary system simplifies for light particle flux:**

$$\rho_i (u_i - u_{cm}) = -\mu \left( \alpha_{11} \left( \nabla \chi + (\chi - Y) \frac{\nabla p_{ia}}{p_{ia}} + (z - Y) \frac{\nabla p_e}{p_{ia}} + \left( z - \frac{Z_i^2 n_i}{\sum_j Z_j^2 n_j} \right) \alpha_{Te} \frac{\nabla T_e}{T_i} \right) + \frac{3}{2} \alpha_{Ti} \frac{\nabla T_i}{T_i} \right)$$

For $\nabla \chi \Rightarrow \frac{D\chi}{DY} \nabla Y$

concentration given as mass fraction (Kagan, Tang, 2014)

concentration and ion barodiffusion terms are equivalent in Zimmerman-Schunk model (Hoffman, et.al., PoP, 2015)

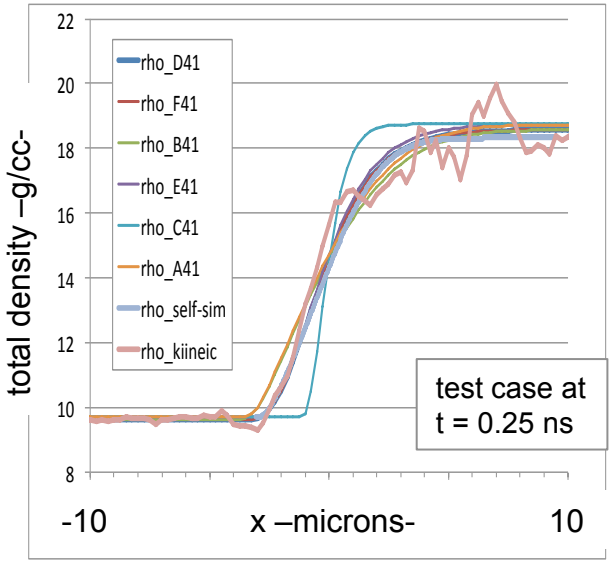
$$\frac{1}{p_{ia}} (\nabla p_i - Y \nabla p_{ia})$$

$$\frac{\nabla \chi}{\nabla Y} = \frac{D\chi}{DY} = \frac{\varepsilon_m}{(Y + (1-Y)\varepsilon_m)^2} = \varepsilon_m (\chi + (1-Y)/\varepsilon_m)^2$$

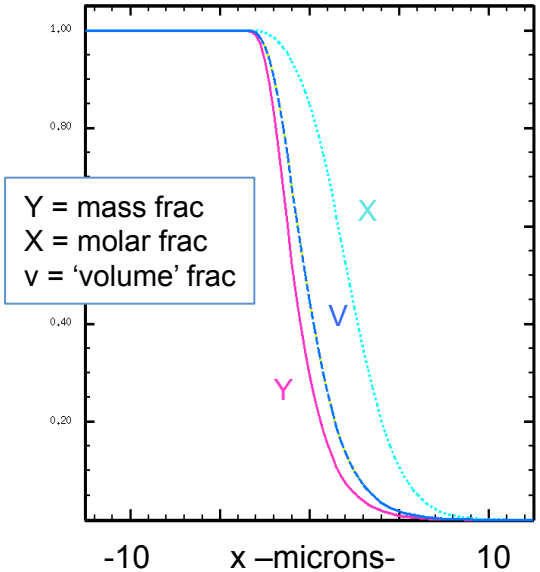
$$\varepsilon_m = m_i / m_I$$

# Plasma transport in xRage (Eulerian AMR code) compares well to non-linear self-similar diffusion test problem[1] under P-T equilib.

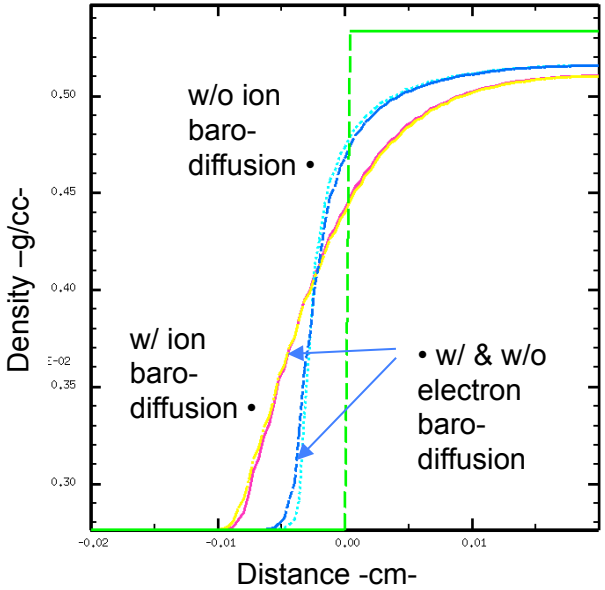
- code solutions during development converged on analytic self-sim soln <u>and</u> kinetic soln for D-Al mixing.




- code profiles for X, Y matches analytics in Molvig, et.al., 2014[1] (for ideal gas or EOS tables )




- mix profile contributions for low z – high z mixing: DD-Au
- ion barodiffusion ~ doubles mix width for DD-Au




- ion barodiffusion contributes less for lower z mixing

[1] D-AL planar mixing at 4 keV, Molvig, Vold, Dodd, Wilks, PRL, 2014

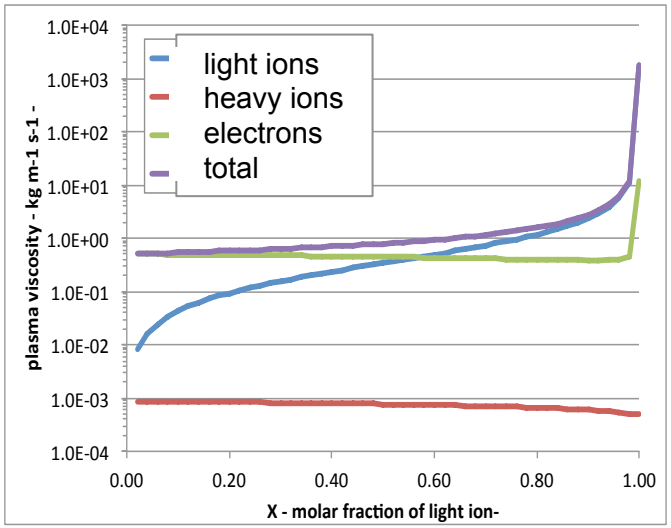
# Viscosity

- **Previous work showed classical binary viscosity can be approximated as a sum over species, and terms simplify considerably in 1D spherical symmetry.**

  - **Ion viscosity in classical binary transport (Molvig, Simikov, Vold, PoP 2014):**

$$\eta_M = m_i n_i \frac{4}{\sqrt{3}} \frac{v_{Ti}^2}{\nu_i[n\ ]} \frac{\alpha_\eta[\Delta_I]}{x_I Z_I^2} = \frac{4}{\sqrt{3}} \frac{m_i v_{Ti}^2}{\nu_{ii}[n_i]} \frac{n_i}{n} \frac{\alpha_\eta[\Delta_I]}{x_I Z_I^2} \propto \frac{x_i \alpha_\eta[\Delta_I]}{x_I Z_I^2} \sim \frac{\alpha_\eta[\Delta_I]}{\Delta_I}$$

$$\Delta_I = \frac{x_I Z_I^2}{x_i Z_i} \approx \frac{(1-x_i)\ Z_I^2}{x_i} = \frac{\varepsilon_m (1-y_i)\ Z_I^2}{y_i}$$




  - **Ion viscosity in approx as species summation**

$$\eta_M \approx kT \sum_i \left[ \frac{n_i}{\nu_i} \right] = kT \sum_i \left[ \frac{n_i}{\sum_j \nu_{ij}} \right] \qquad \eta_o \approx \frac{n_i k T_i}{\nu_i} \approx \frac{n_i k T_i}{\sum_j \nu_{i,j}} \approx \frac{n_i k T_i}{C_\nu(\nu_{1,1} + \nu_{1,2})} \approx \frac{n_i k T_i^{5/2}}{C_\nu(n_1 + n_2 z_2^2)}$$

$$\nu_{ij} = C_\nu \alpha_{ij}^K \frac{z_i^2 z_j^2 L_{ij} n_j}{m_i^{1/2} k T^{3/2}}$$

Plasma viscosity due to electrons is important in high z material. ( X = 0 here)

  - **Viscous terms simplify in 1D spherical geometry**
    - **viscous tensor:**

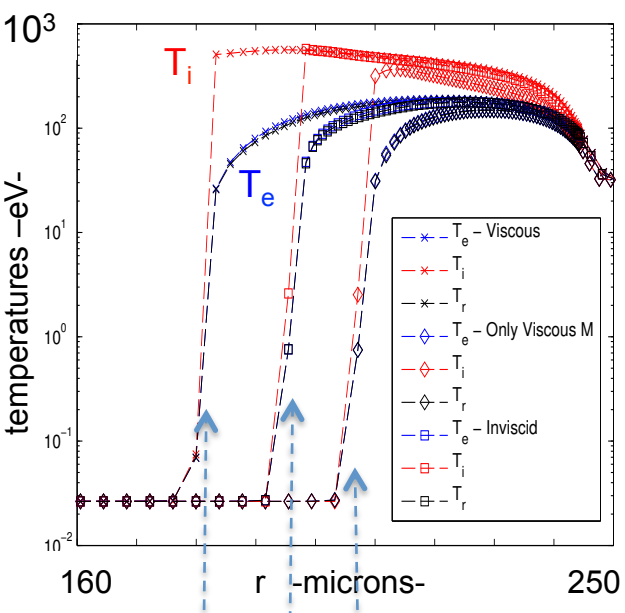
$$(\nabla : \tau)_r [1D] = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{4}{3} \eta_o \left( \frac{\partial u}{\partial r} - \frac{u}{r} \right) \right) + \frac{4\eta_o}{3r} \left( \frac{\partial u}{\partial r} - \frac{u}{r} \right)$$

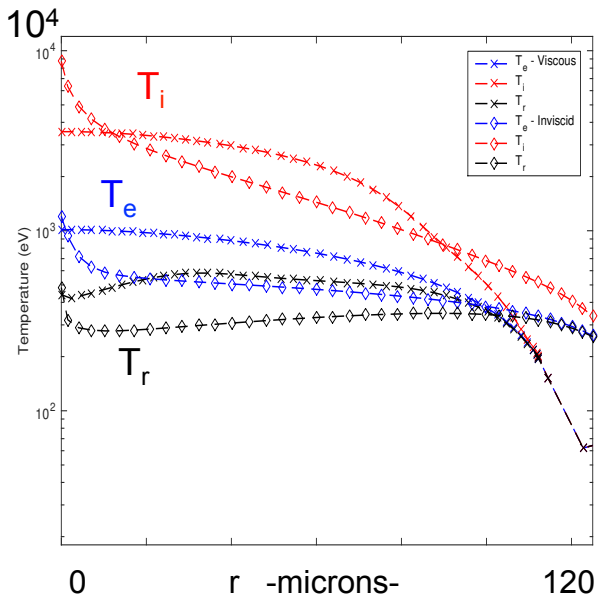
    - **viscous energy dissipation**

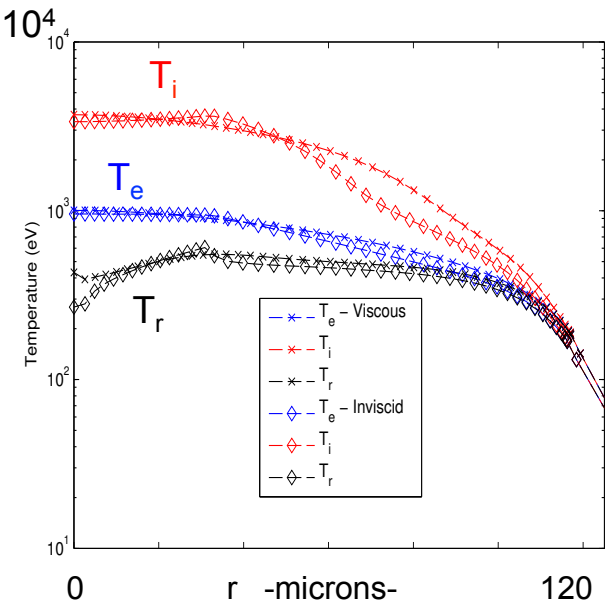
$$\Phi_{visc} = \eta_o \left( 2 \left( \left( \frac{\partial u}{\partial r} \right)^2 + 2 \left( \frac{u}{r} \right)^2 \right) - \frac{2}{3} \left( \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 u) \right)^2 \right)$$

# Temperature profiles from 1D Lagrange simulations are sensitive to viscous terms and at center show a peak in $T_i$ for the inviscid case.

- **during incoming shock convergence**
- **at shock convergence**
  - **(1.42 ns inviscid)**
  - **(1.52 ns viscous)**
- **after shock convergence**
  - **(1.56 ns )**



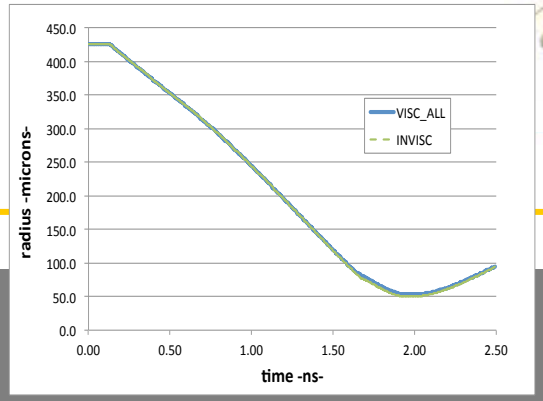

plasma viscosity only in momentum equation

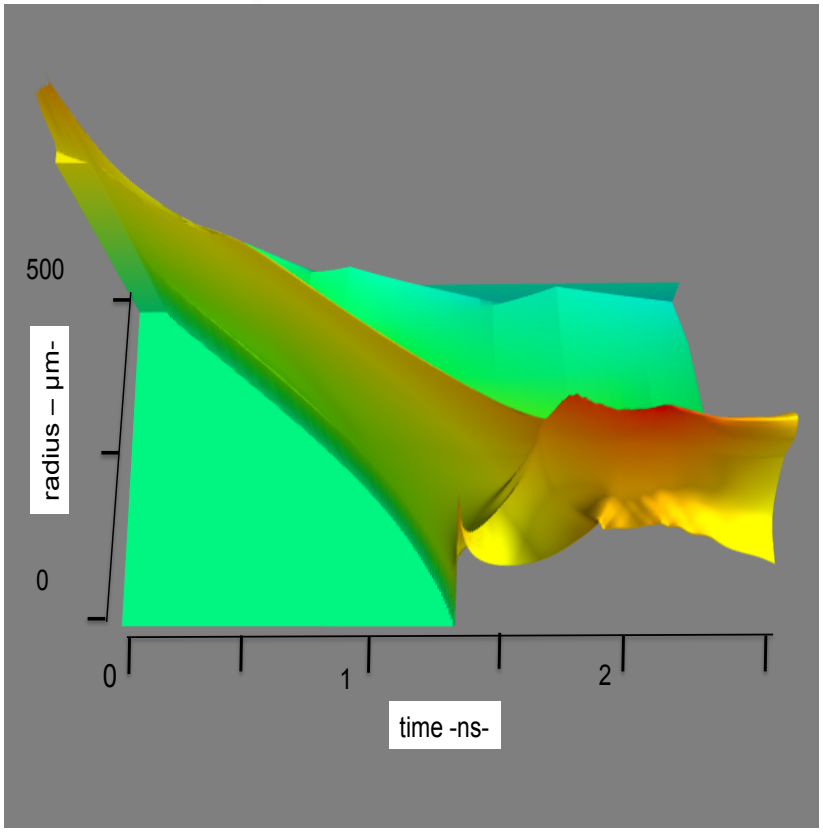
inviscid

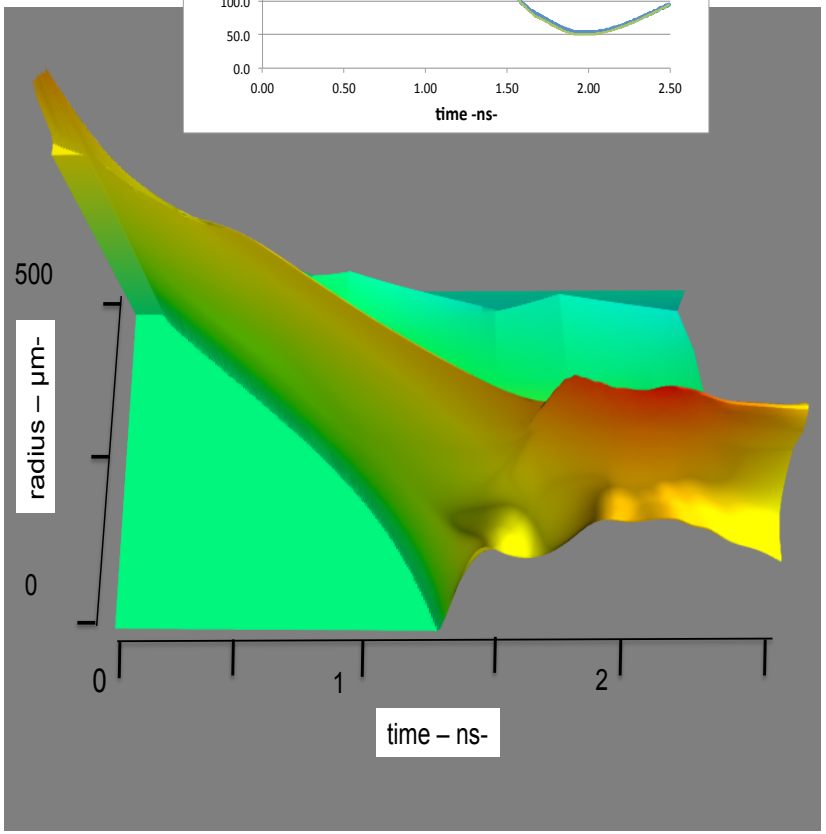
plasma viscosity

Inviscid case shows Ti peak on center at shock convergence and rapid relaxation.

# Density [r,t] surface in a 1D Lagange code: ICF implosion with and without plasma viscosity

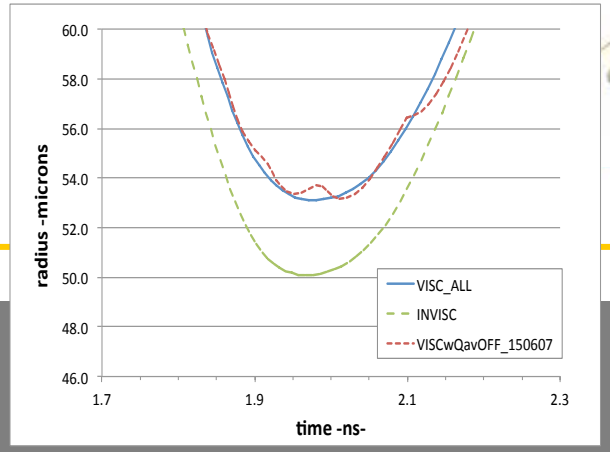


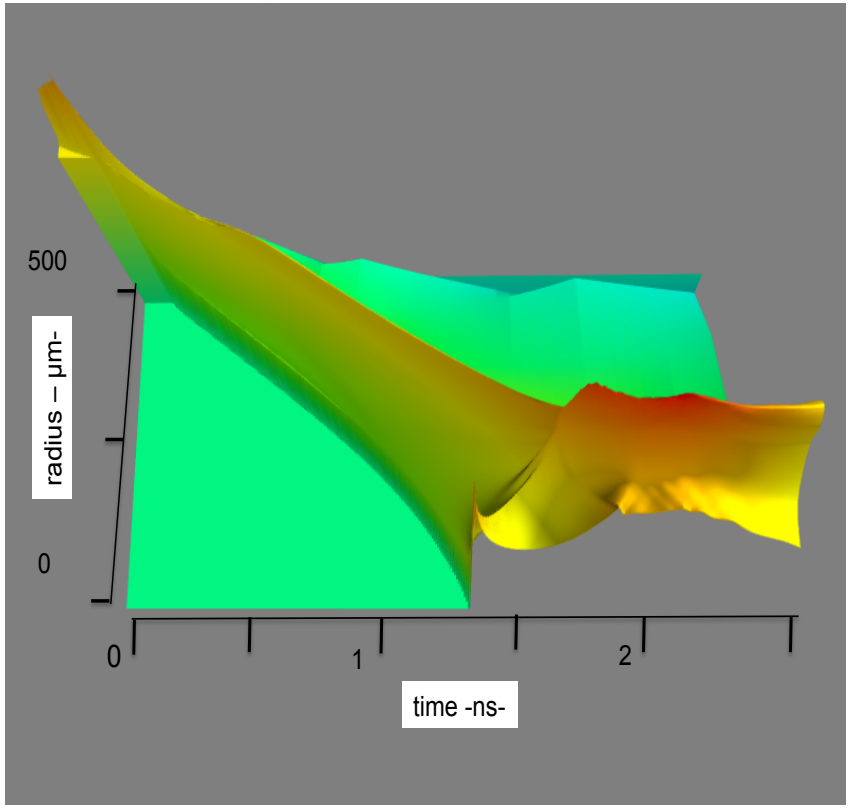



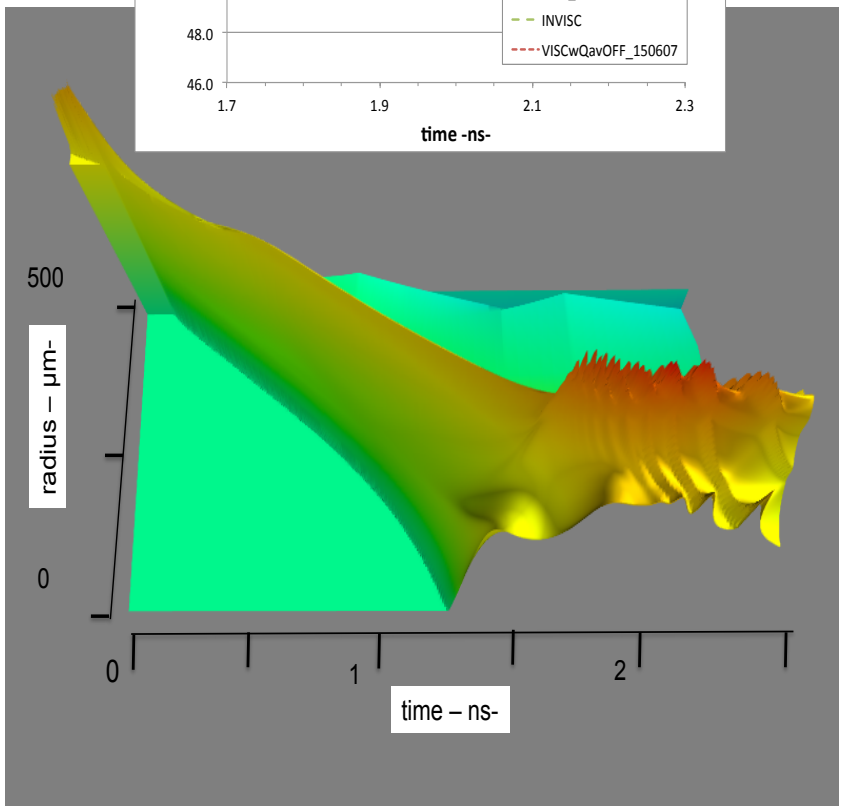




- ‘Standard’ Lagrange w/ artificial viscosity
- Lagrange w/ plasma and artificial viscosity

# Density [r,t] surface in ICF implosion with and without plasma viscosity AND w/ 'late time' artificial viscosity zeroed




- 'Standard' Lagrange w/ artificial viscosity
- Lagrange w/ plasma viscosity
- NO artificial viscosity (after shock convergence)

# Kinetic coefficients**, for ion temperature gradient drive in binary species mixing, are of order unity when using molar concentrations but appear large using mass concentrations.

- **Assume (to simplify):**
- **Y (mass frac) = 0.5 Erfc[ξ]**
- **$\xi = z / L_D = z / (2(D\,t)^{1/2})$**

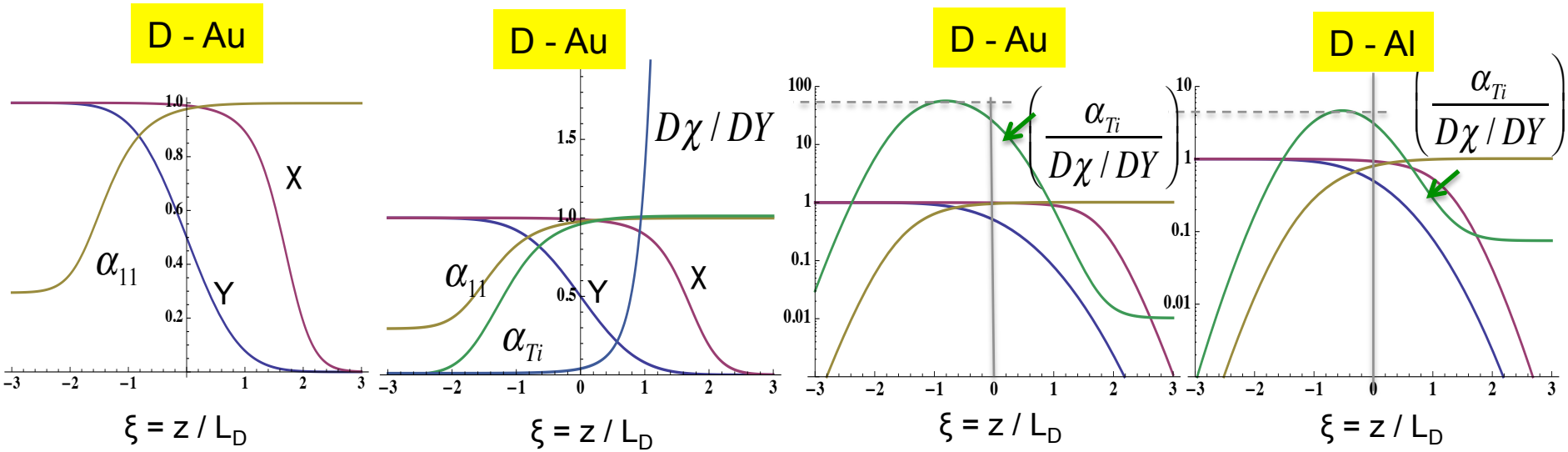
$$\rho_i(u_i - u_{cm}) = -\mu\left(\alpha_{11}\left(\nabla\chi + (\chi - Y)\frac{\nabla p_i}{p_i} + (z - Y)\frac{\nabla p_e}{p_i} + \left(z - \frac{Z_i^2 n_i}{\sum_j Z_j^2 n_j}\right)\alpha_{Te}\frac{\nabla T_e}{T_i}\right) + \frac{3}{2}\alpha_{Ti}\frac{\nabla T_i}{T_i}\right)$$

$$\nabla\chi = \left(D\chi / DY\right)\nabla Y$$

$$\chi_i = \frac{n_i}{n_i + n_I} = \frac{\rho_i / m_i}{\rho_i / m_i + \rho_I / m_I} = \frac{y_i}{y_i + (1 - y_i)\varepsilon_m}$$

$$\frac{\nabla\chi}{\nabla y} = \frac{D\chi}{Dy} = \frac{\varepsilon_m}{(y + (1-y)\varepsilon_m)^2} = \varepsilon_m(\chi + (1-\chi)/\varepsilon_m)^2$$

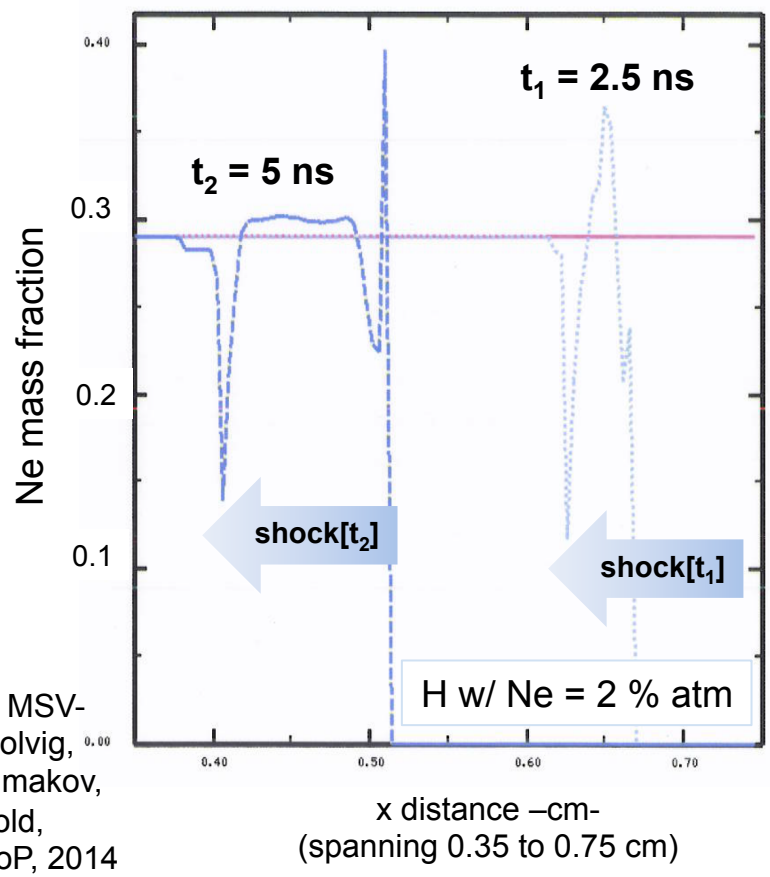
$$\varepsilon_m = m_i / m_I$$



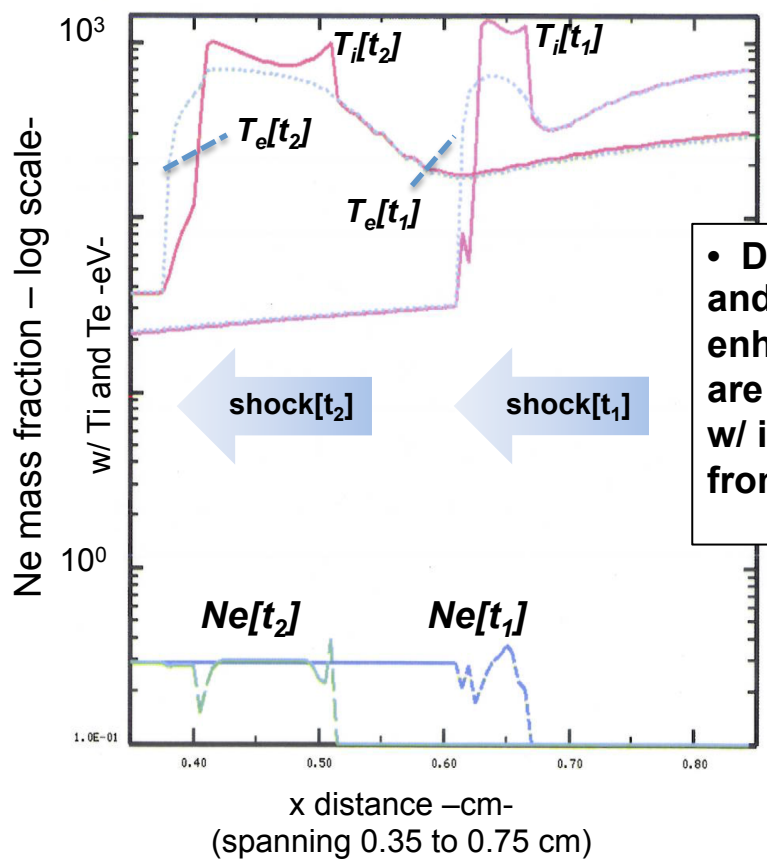

** Molvig, Simikov, Vold, PoP, 2014

$\left(\frac{\alpha_{Ti}}{D\chi / DY}\right)$ = grad[Ti] coef. relative to mass fraction gradient (e.g., Kagan-Tang form)

**Preliminary simulations (xRage) of Rinderknect planar shock experiment using MSV** ion transport model show a depletion of Ne at the ion shock front followed by a small enhancement of Ne concentration behind the shock front.**



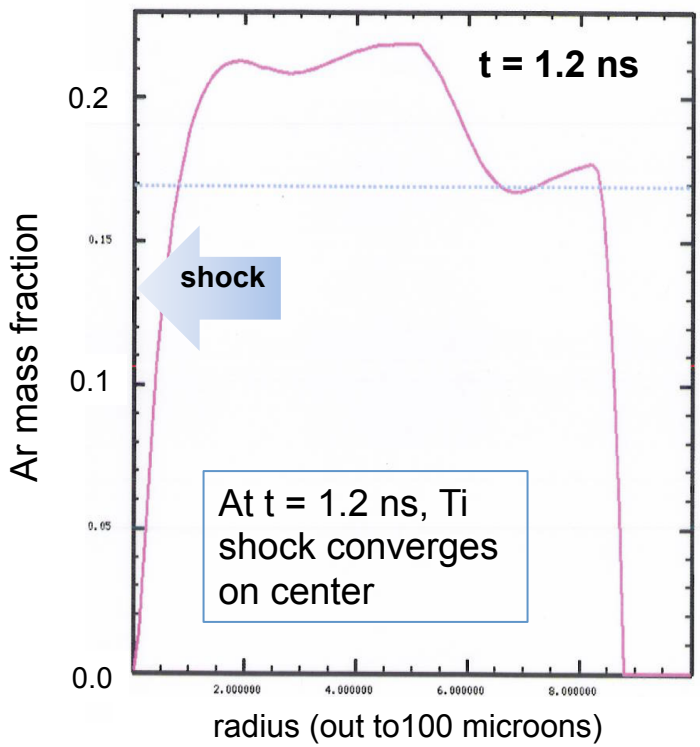

** MSV-Molvig, Simakov, Vold, PoP, 2014

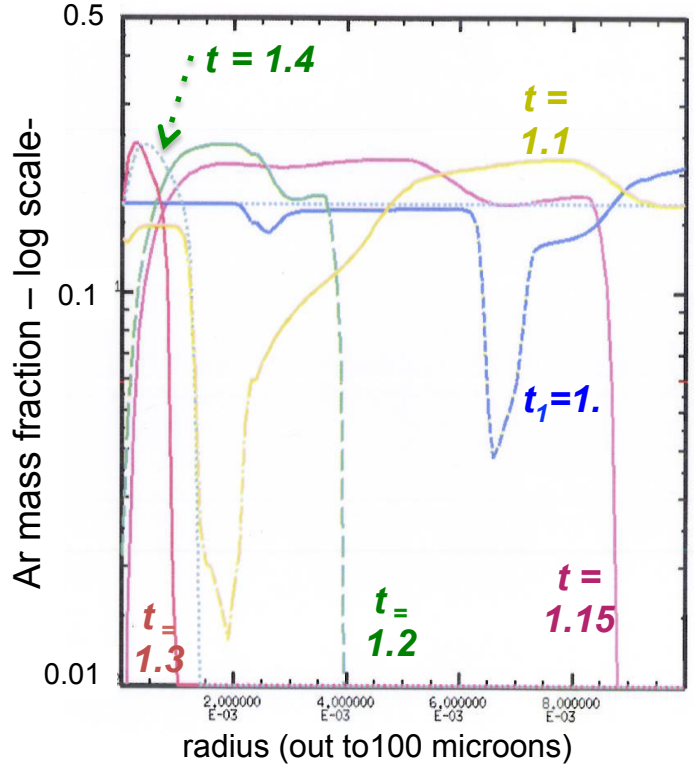



- **Depletion and enhancement are persistant w/ ion shock front.**

- **Depletion/Enhancement is a result of ion thermodiffusion.**

# Simulations (xRage) of OMEGA shot #78199 using MSV multispecies ion-transport model show a depletion of Ar at the incoming ion shock front followed by a persistent enhancement of argon concentration behind the shock and following shock reflection at center.




- Depletion and Enhancement persist for many 100s of ps.
- Bang time ~ 1.35 ns
- times (ns) are:

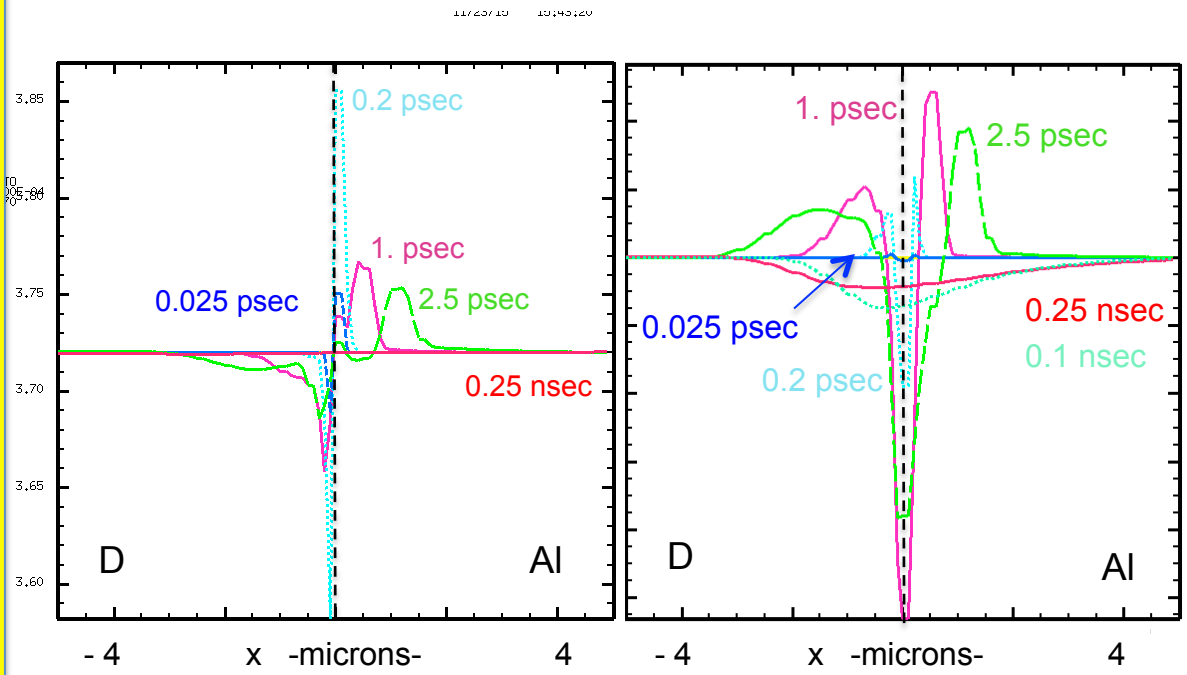
1., 1.1, 1.15, 1.2, 1.3, 1.4

- **Enhancement is a result of ion thermodiffusion**
  - Simulations with ion thermodiffusion turned off do not show this effect.

# Plasma transport simulations (xRage) showing mix layer profile evolution at early times (< ~ psec, Kn ~ mfp/$L_D$ large) and into fluid regime ( ~ ns, Kn ~ mfp/$L_D$ small) for DD-Al mixing at 4 keV).

**3 distinct phases:**
1 - << 1 psec: pressure discontinuity, dP, grows at interface.
2 - ~ 1- 2 psec: dP propagates into each fluid at its sonic speed.
3- < 0.1-0.2 ns: dP has relaxed -> 0, in the mix region

- pressure
- velocity




**3 distinct phases:**
1 - << 1 psec: u grows starting 'tri-model' profile
2 - ~ 1- 2 psec: u tri-model profile relaxes in magnitude, spreading in space
3- < 0.1-0.2 ns: dP relaxed -> ~ 0, while div*u ≠ 0, and relaxes on diffusion time scale

••• Results are being compared to kinetic simulations in VPIC (L.Yin) and iFP (Chacon, Taitano) to understand early time (large Kn number) mix behavior.

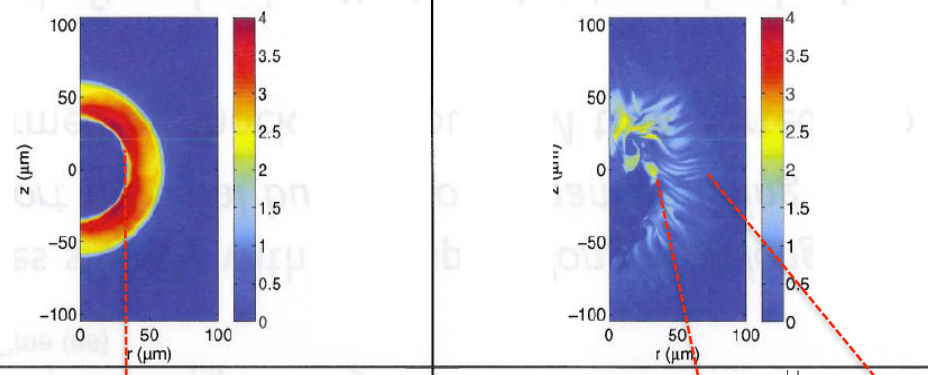
# Small scale structures: CAUTION: numerics can mask the plasma transport mixing

- **Numerical solutions for ICF 'sym-cap' mixing** show detailed structure w/ more realistic IC.**
- **When structure sizes are comparable to numerical diffusion scales, the plasma diffusion may not greatly modify the solution.**

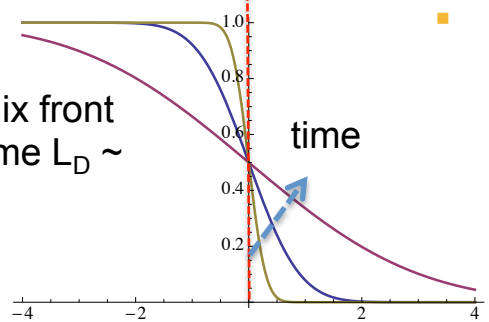
simple IC geometry | 'detailed' IC geometry

No stalk, beam imbalances (2D sim.) | With stalk, beam imbalances (2D slice)

$\rho_{CD}$ (g/cm$^3$)

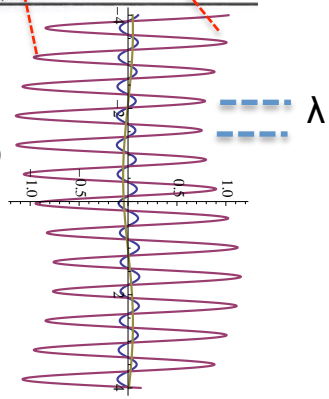



- diffusion mix front grows in time $L_D \sim 2(D\,t)^{1/2}$




'side' view

- For RM, RT, KH instabilities, mix volume related to interfacial area, A, in more complex manner




'top' view

** *Haines, Grimm, et.al. in preparation, 2016*

mix volumes: early time: $V_m[t] \sim A_m[t]\, L_D[t]$ .....and finally… $V_m[t] \sim f[\, A_m[t], L_D[t], \lambda[t]\, ]$